Exercise 41

Evaluate the integral.

$$\int_0^4 2^s \, ds$$

Solution

The goal is to write the integrand as a derivative of some function. Write the integrand in terms of base e rather than base 2 by using the corresponding logarithm.

$$2^s = e^{\ln 2^s} = e^{s \ln 2}$$

Taking the derivative of both sides yields a factor of $\ln 2$ because of the chain rule.

$$\frac{d}{ds}(2^s) = \frac{d}{ds}(e^{s \ln 2}) = (\ln 2)e^{s \ln 2}$$

Divide both sides by $\ln 2$.

$$\frac{1}{\ln 2}\frac{d}{ds}(2^s) = e^{s \ln 2}$$

Consequently,

$$\frac{1}{\ln 2}\frac{d}{ds}(2^s) = 2^s.$$

Therefore, using the second part of the fundamental theorem of calculus,

$$\begin{aligned}
\int_0^4 2^s \, ds &= \int_0^4 \frac{1}{\ln 2}\frac{d}{ds}(2^s) \, ds \\
&= \frac{1}{\ln 2}\int_0^4 \frac{d}{ds}(2^s) \, ds \\
&= \frac{1}{\ln 2}(2^s)\Big|_0^4 \\
&= \frac{1}{\ln 2}(2^4 - 2^0) \\
&= \frac{1}{\ln 2}(16 - 1) \\
&= \frac{15}{\ln 2}.
\end{aligned}$$